

# How Feynman tried to quantize gravity and discovered ghosts, and other stories

Hirosi Ooguri

Aspen Center for Physics Colloquium  
23 August 2018

# How Feynman tried to quantize gravity

*"There's a certain irrationality to any work in gravitation, so it's hard to explain why you do any of it."*

based on a tape-recording of Feynman's lecture "Quantum Theory of Gravitation" at the Conference on Relativistic Theories of Gravitation organized by Leopold Infeld in Poland in July 1962.



Dirac and Feynman  
in the Jabłonna Palace  
near Warsaw, Poland  
in July 1962.



In the 1962 lecture in Poland, Feynman presented what is now known as the **Feynman Tree Theorem**.

In quantum theory, probabilities of all possible outcomes always sum up to be one.

From this assumption, Feynman derived a remarkable set of **relations between classical amplitudes and quantum amplitudes** in non-gravitational theory.

*tree = classical*



Feynman explaining his tree theorem  
in Jabłonna, Poland in July 1962.

# Feynman Tree Theorem

Relations between classical and quantum amplitudes

*"I assumed the tree theorem to be true, and used it in reverse."*

He discovers that **these relations do not hold in gravity** with virtual gravitons in quantum processes.

*"something is fundamentally wrong"*

based on a tape-recording of Feynman's lecture  
"Quantum Theory of Gravitation" in Poland in July 1962.

Gell-Mann suggested him to study the Yang-Mills theory, and Feynman found the same problem.

*"if you have two examples of the same disease, then there are many things you don't worry about."*

*"The Yang-Mills case took me about a day."*

However, the gravitational case was harder.

*"I tried again and again and was never able to do it; and it was finally put on a computing machine. ... I think it's the first problem in algebra that I know of that was done on a machine that has not been done by hand."*

based on a tape-recording of Feynman's lecture  
"Quantum Theory of Gravitation" in Poland in July 1962.

Feynman noticed that his tree theorem also holds in the gravitational case if an "artificial dopey particle" is included in virtual quantum processes.

*" I only got this completely straightened out a week before I came here. I do not claim that this method of quantization can be obviously and evidently carried out on to the next order. "*

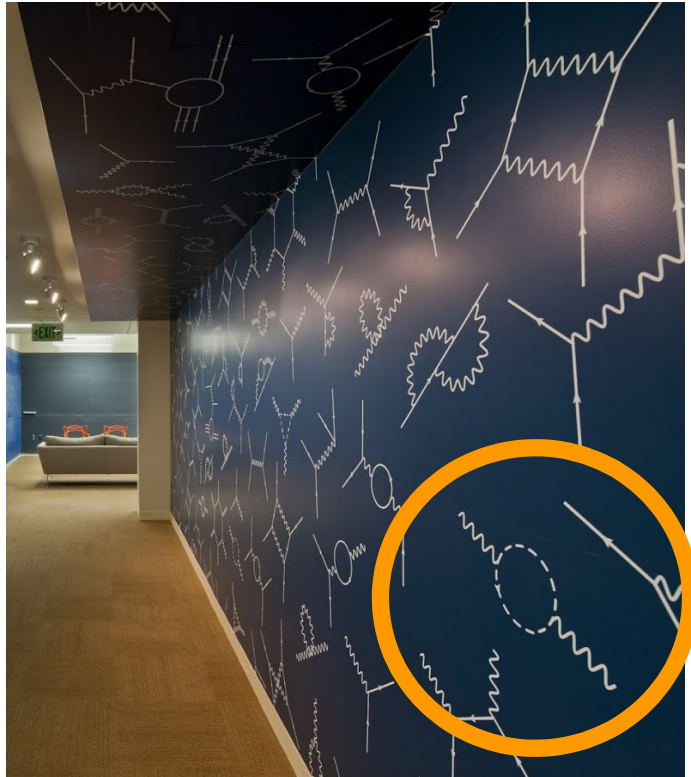
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**His "dopey particle" proposal has turned out to be essential in quantizing Yang-Mills and gravity.**

based on a tape-recording of Feynman's lecture  
"Quantum Theory of Gravitation" in Poland in July 1962.



- 1967 De Witt, "Quantum Theory of Gravity. I, II, III"  
Faddeev and Popov, "Feynman Diagrams for the Yang-Mills Field"
- 1972 't Hooft and Veltman, "Regularization and Renormalization of Gauge Fields"
- 1973 Gross and Wilczek, "Ultraviolet Behavior of Nonabelian Gauge Theories"  
Politzer, "Reliable Perturbative Results for Strong Interactions?"





# Variations on a Theme of Feynman Tree Theorem

Review

## On-shell methods in perturbative QCD

Zvi Bern<sup>a,1</sup>, Lance J. Dixon<sup>b,\*,2</sup>, David A. Kosower<sup>c,3</sup>

<sup>a</sup> *Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095-1547, USA*

<sup>b</sup> *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*

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Received 7 April 2007; accepted 7 April 2007

Available online 18 April 2007

## The All-Loop Integrand For Scattering Amplitudes in Planar $\mathcal{N} = 4$ SYM

N. Arkani-Hamed<sup>a</sup>, J. Bourjaily<sup>a,b</sup>, F. Cachazo<sup>a,c</sup>, S. Caron-Huot<sup>a</sup>, J. Trnka<sup>a,b</sup>

<sup>a</sup> *School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA*

<sup>b</sup> *Department of Physics, Princeton University, Princeton, NJ 08544, USA*

<sup>c</sup> *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2J W29, CA*

### Abstract

We give an explicit recursive formula for the all  $\ell$ -loop integrand for scattering amplitudes in  $\mathcal{N} = 4$  SYM in the planar limit, manifesting the full Yangian symmetry of the theory. This generalizes the BCFW recursion relation for tree amplitudes to all loop orders, and extends the Grassmannian duality for leading singularities to the full amplitude. It also provides a new physical picture for the meaning of loops, associated with canonical operations for removing particles in a Yangian-invariant way. Loop amplitudes arise from the “entangled” removal of

---

### Abstract

We review on-shell methods for computing multi-parton scattering amplitudes in perturbative QCD, utilizing their unitarity and factorization properties. We focus on aspects which are useful for the construction of one-loop amplitudes needed for phenomenological studies at the Large Hadron Collider.

PRL **94**, 181602 (2005)

PHYSICAL REVIEW LETTERS

week ending  
13 MAY 2005

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## Direct Proof of the Tree-Level Scattering Amplitude Recursion Relation in Yang-Mills Theory

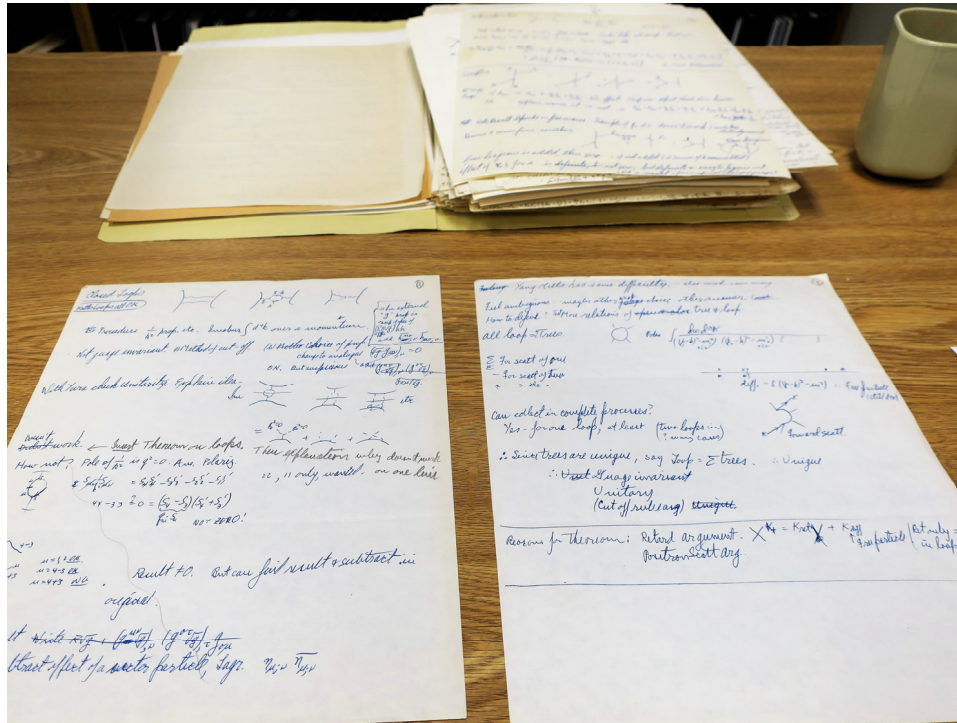
Ruth Britto, Freddy Cachazo, Bo Feng, and Edward Witten

*School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540, USA*

(Received 14 February 2005; published 10 May 2005)

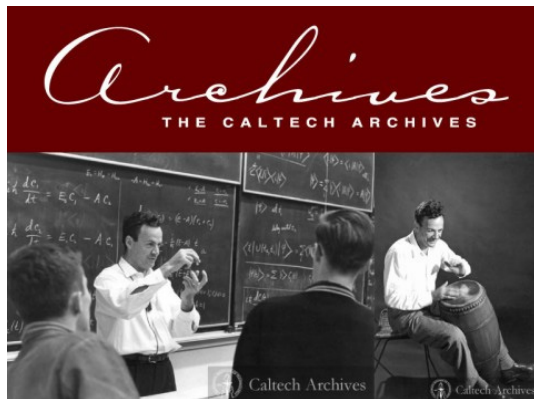
Recently, by using the known structure of one-loop scattering amplitudes for gluons in Yang-Mills theory, a recursion relation for tree-level scattering amplitudes has been deduced. Here, we give a short and direct proof of this recursion relation based on properties of tree-level amplitudes only.





Feynman's notes on gravitation in one of the 93 Feynman boxes at the Caltech Archives.

This folder also contains a manuscript for his talk at the 1962 conference in Poland.



**Feynman Exhibition**  
(May 11 - December 14)



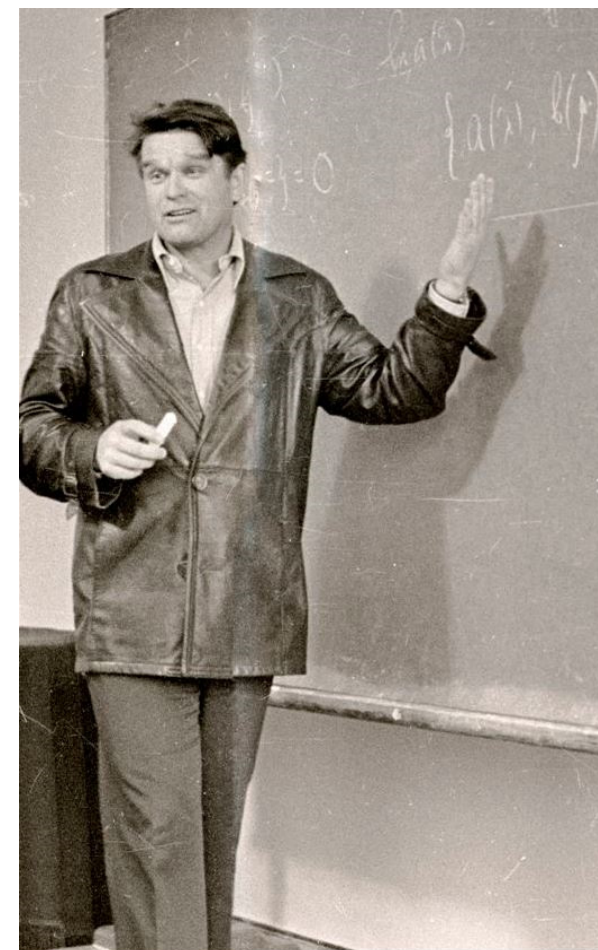
To prove that younger generation knows and respects Feynman Integral  
L. Faddeev

## FEYNMAN DIAGRAMS FOR THE YANG-MILLS FIELD

L. D. FADDEEV and V. N. POPOV  
*Mathematical Institute, Leningrad, USSR*

Received 1 June 1967

Feynman and De Witt showed, that the rules must be changed for the calculation of contributions from diagrams with closed loops in the theory of gauge invariant fields. They suggested also a specific recipe for the case of one loop. In this letter we propose a simple method for calculation of the contribution from arbitrary diagrams. The method of Feynman functional integration is used.



A young theorist sent his annotated paper to Feynman.



$d\Omega$  - invariant measure on the group.

ymptote at  $t = x_0 \rightarrow \pm\infty$  prescribed by in-and out-states. The diagrams appear naturally in the perturbative calculation of this integral.

In the case of gauge invariant theory it is necessary to transform this integral a little. In fact, we can say, using the natural geometrical language, that the integrand is constant on the "orbits"  $B_\mu \rightarrow B_\mu^g$  of the gauge group in the manifold of all fields  $B_\mu(x)$ . It follows that the integral itself is proportional to the volume of this orbit which can be expressed as the integral  $\int \prod_x [d\Omega(x)]$

over all matrices  $\Omega(x)$ . This integral should be factorized before using the perturbation theory.

$$\int dx_1 \dots \int dx_n \text{Sp}(B^{\mu_1}(x_1) \dots B^{\mu_n}(x_n)) \times \partial_{\mu_1} G(x_1 - x_2) \dots \partial_{\mu_n} G(x_n - x_1)$$

where  $G(x)$  is a Green function of the D'Alembert operator. This expression corresponds to the closed loop with the scalar particle propagating along it and interacting with the transverse vector particles according to the law  $\sim \epsilon \text{Sp}(\phi [B^\mu \partial_\mu \phi])$ .

There results the diagram technique with the following features:

1. The pure transversal Green function is to

Formally  $\Delta[B]$  is equal to the determinant of the operator

$$Au = \square u - \epsilon [B^\mu, \partial_\mu u] = A_0 u - \epsilon V(B)u$$

After extracting the trivial infinite factor  $\det A_0$  we obtain the following expression for  $\ln \Delta[B]$

$$\ln \Delta[B] = \ln(\det A / \det A_0) = \text{Sp} \ln(1 - \epsilon A_0^{-1} V(B))$$

Developing the right hand side in a power series in  $\epsilon$  we have the following expressions for the coefficients

30

can be written as a Feynman integral over anticommuting variables  $u, \bar{u}$ :

$$\Delta[B] = \int \exp \left\{ \frac{i}{2} \int_x \bar{u} \cdot \square u - \epsilon (\bar{u} [B_\mu \cdot \partial_\mu u]) \right\} \prod_x du d\bar{u}$$

coordinate system.

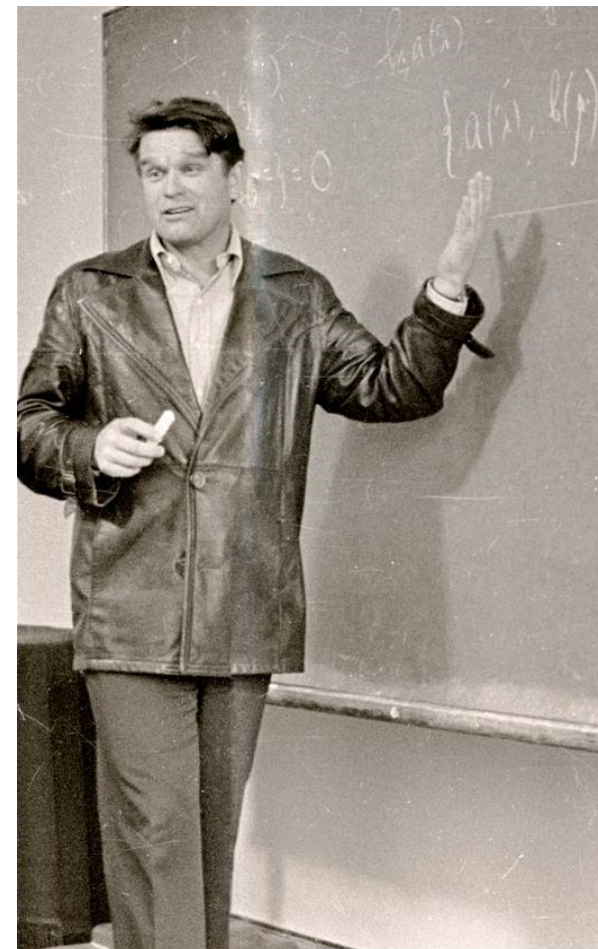
References

1. C. N. Yang and R. L. Mills, Phys. Rev. 96 (1954) 191.
2. R. Utiyama, Phys. Rev. 101 (1956) 1597.
3. S. L. Glashow and M. Gell-Mann, Ann. of Phys. 15 (1961) 437.
4. R. P. Feynman, Acta Physica Polonica, 24 (1963) 697.
5. B. S. De Witt, Relativity, groups and topology. (Blackie and Son Ltd 1964) pp 587-820.
6. R. P. Feynman, Phys. Rev. 80 (1950) 440.

Miraculously one can use

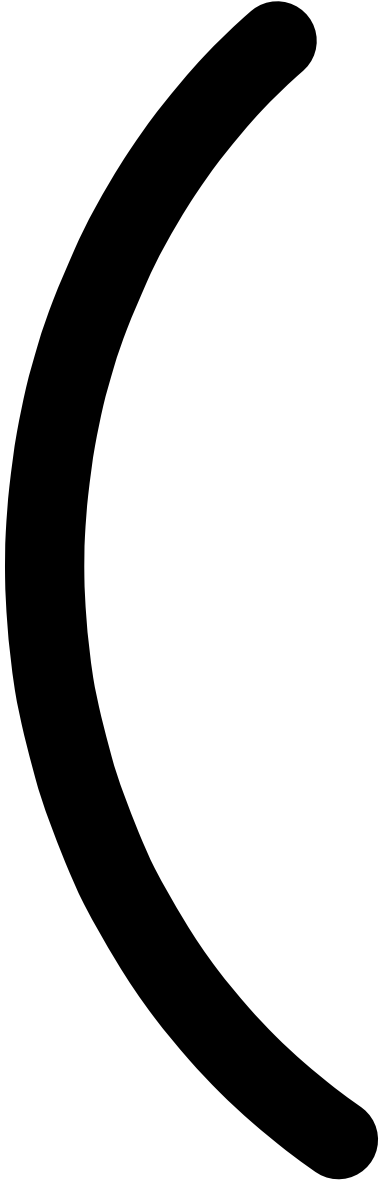
$\Delta[B]$  here instead of "cumbersome"  $\epsilon[B]$ , i.e.

"stationary phase" answer for  $\epsilon[B]$  can be used as being exact.



A young theorist sent his annotated paper to Feynman.

Begin a digression





Bamberg, Germany (March 2008)

Ludwig Dimitrevich Faddeev:

*"I saw a huge folder of Feynman's notes on the **Bethe ansatz**.*

*I would like to know why he was interested in integrable models.*

*Could you find the notes?"*

**Ansatz:** an attempt at a solution followed by a demonstration that it is correct, rather than a solution derived using a known approach.

For the Heisenberg model of  $L$  spin in a 1D circle, **Hans Bethe** (1931) started with the ferromagnetic ground state and considered a subspace with  $n$  spins overturned. Going to the momentum basis,

$$\psi(x_1, \dots, x_n) = \sum_{\pi \in \Theta_n} A_\pi \exp(i k_{\pi_1} x_1 + \dots + i k_{\pi_n} x_n),$$

Bethe assumed,

$$A_\pi = (-1)^\pi \prod_{i < j} S_{\pi_i \pi_j}.$$

The momenta then must satisfy,

$$(-1)^{n-1} e^{i k_j L} = \prod_{l \neq j} \frac{S_{lj}}{S_{jl}}$$

This gives  $2^L$  energy eigenstates.



Feynman's blackboard when he passed away in February, 1988.

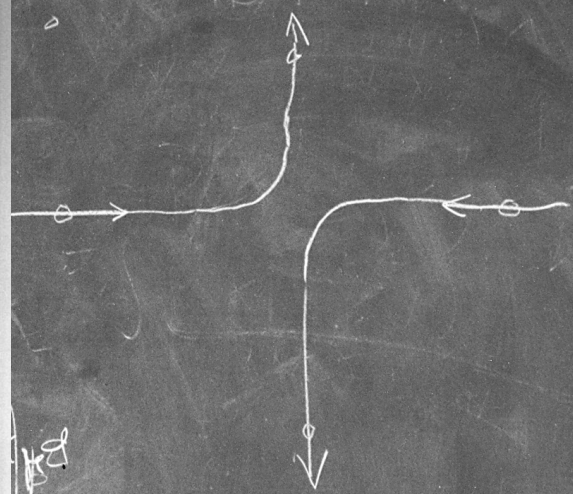
What I cannot create,  
I do not understand.

Know how to solve every  
problem that has been solved

Why const  $\times$  SORT .POM

TO LEARN:  
 Bethe Ansatz Probs.  
 Kondo  
 2-D Hall  
 accel. Temp  
 Non linear Classical Hydro

(A)  $f = U(r, a)$   
 $g = 4(r, z) u(r, z)$   
 (B)  $f = 2|r, a| u(r, a)$



QUANTUM MECH REVERSIBILITY (PROB. INTEGRAL)

$M = WE + a + N$      $[M, W] = -i\hbar(L+1)$

$\beta = \frac{c}{g^2} \leftarrow a$

$\infty^3$      $T =$      $\psi(x)$

$U =$      $W =$      $L = U(t)WU(t)$

$P^+ = -P$      $P f(x) = f(-x) \rightarrow \text{POL}$      $L = \dot{U}WU - UW\dot{U}$

$\int_{x_1}^{x_2} f(x) dx = -\ln x$      $\dot{L} = \dot{U}UL - L\dot{U}$

$x(t) \psi^{(n)}(t) = M, \sum P_n, I_n, J_n, P_n, X_n$      $\dot{x} = f(x) \frac{dx}{dt} = A$      $\dot{U} = AU$

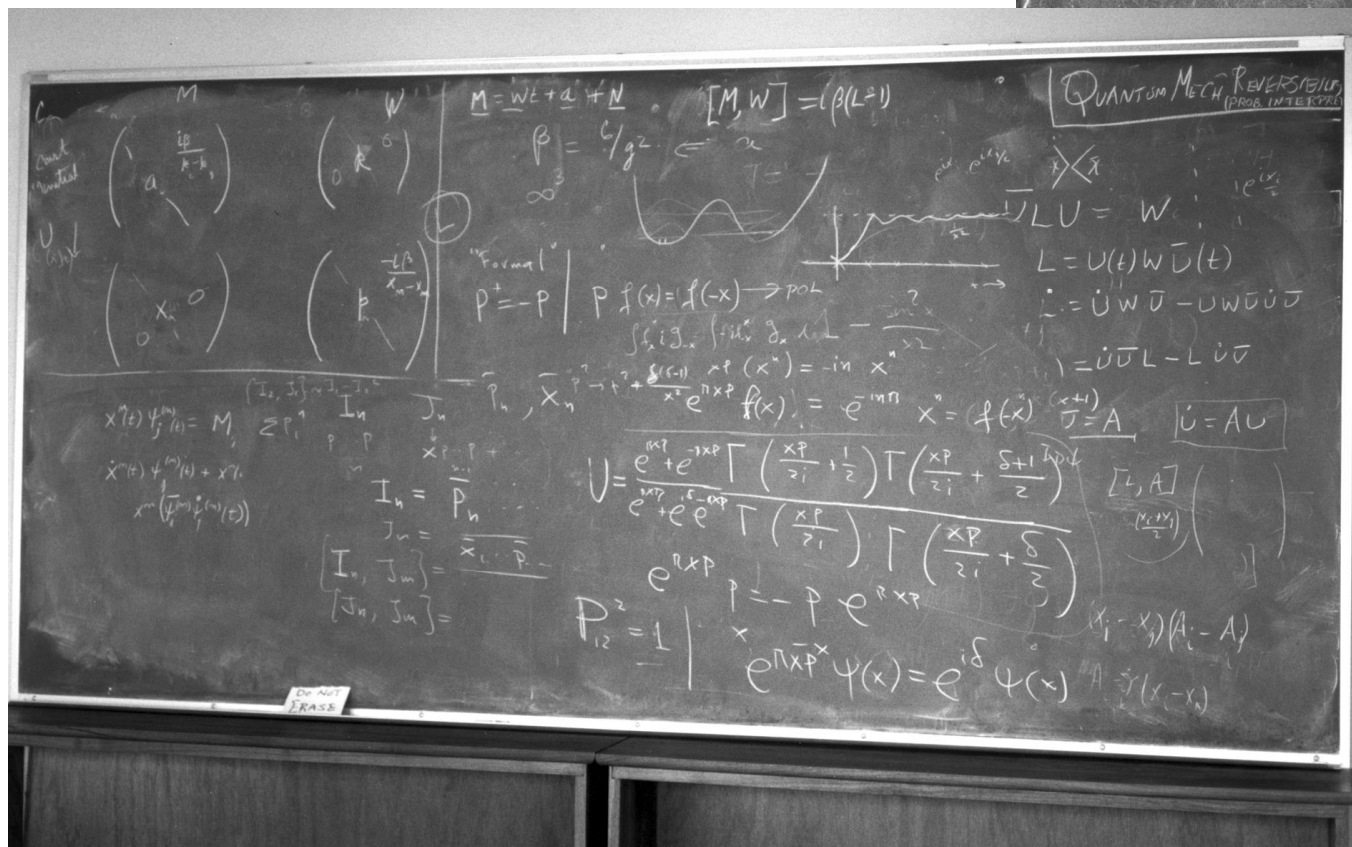
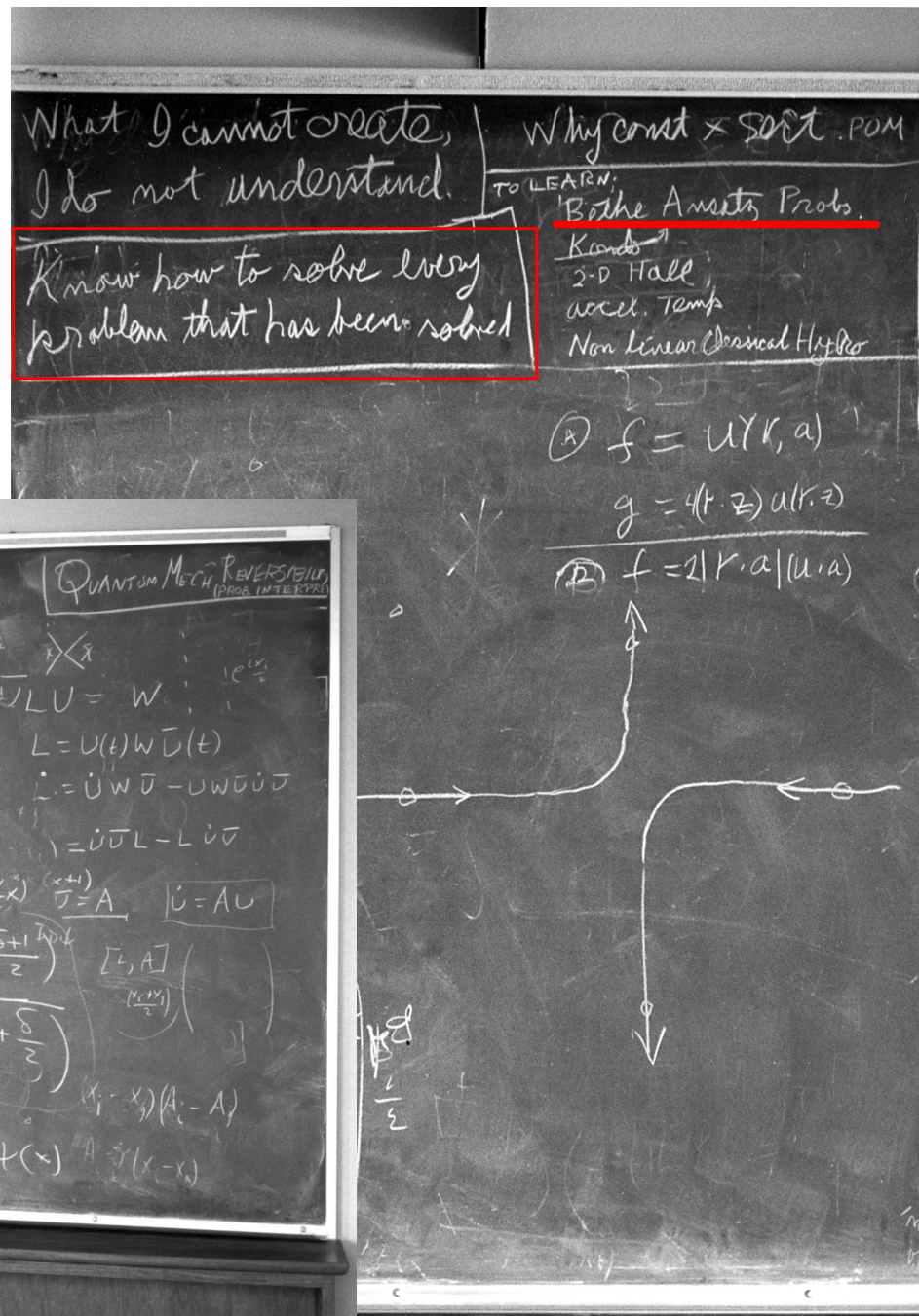
$\dot{x}^{(n)}(t) \psi^{(n)}(t) + x^{(n)}$      $I_n = \frac{P_n}{x_n}$      $J_n = \frac{P_n}{x_n}$

$\{I_n, J_m\} = \frac{P_n P_m}{x_n x_m}$      $\{J_n, J_m\} =$

$P_{12} = 1$      $P = -P e^{RXP}$      $e^{RXP} \psi(x) = e^{i\delta} \psi(x)$

$\Gamma\left(\frac{xP}{2i} + \frac{1}{2}\right) \Gamma\left(\frac{xP}{2i} + \frac{\delta+1}{2}\right)$      $\Gamma\left(\frac{xP}{2i}\right) \Gamma\left(\frac{xP}{2i} + \frac{\delta}{2}\right)$

Feynman's blackboard when he passed away in February, 1988.





Feynman:

*"I got really fascinated by these (1 + 1)-dimensional models that are solved by the Bethe ansatz and how mysteriously they jump at you and work and you don't know why. I am trying to understand all this better."*

Asia-Pacific Physics News (June/July 1988),  
published posthumously.

# 10 page memo for his lunch time talk on the Bethe ansatz in January, 1987.

1/22/87  
Lunch Talk

Bethe Ansatz Thacker Rev. Mod. Phys. 53 2; p253 (1981)

From time to time many different <sup>top</sup> dimensional (x,t) field theories have been proposed as models to learn from. Group one in a while they surprisingly are solved.

Now lowest School = Yang Yang (74) + d, d+1 + with color,  $\lambda$   
 Theorist 12, 17 -  $\int \mathcal{L}(\psi, \psi^\dagger, \partial \psi, \partial \psi^\dagger) - \text{const}$  Spin Group (20) - const  
 Grassmann 12, 17 -  $\int \mathcal{L}(\psi, \psi^\dagger, \partial \psi, \partial \psi^\dagger)$  } running coupling const.  
 $\Sigma: (2, 1) \text{ but } \mathcal{L} = 1$

All sort of two dimensional stat mech emerge Baxter.

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All sort of two dimensional stat mech emerge Baxter.

All solved by same method - guess as to form of wave fun. Bethe Ansatz  
 Bethe 1931 spin waves. OTHER METHODS to solve

When will it work? Zamolodchikov & T. ansatz  
 Classical, Solitons. Fermion & model N=4

Why study? (1) QED & formulation of field theory running coupling const.  
 (2) Not too useful in other examples Kondo  
 (3) Know how to solve every problem which has been solved.  
 (4) Fun.

What is Bethe Ansatz?  
 Spin waves  $H = -\frac{1}{2} \sum \sigma_x \sigma_x' + \sigma_y \sigma_y' + \Delta \sigma_z \sigma_z'$   $N = \sum \sigma_z$  is count of particles  
 $h = -\sum (\sigma_x \sigma_x' + \sigma_y \sigma_y' + \frac{\Delta}{2} (\sigma_z \sigma_z' - 1))$   
 does nothing two spins same, else  
 $h \alpha \beta = (-1) \beta \alpha + \Delta (\alpha \beta)$

All  $\alpha, \beta = 0$   
 One  $\beta$ . If at  $x$  and  $C_x$   $E C_x = -C_{x+1} - C_{x-1} + 2 \Delta C_x$   $C_x = e^{ikx}$   
 Two  $\beta$ .  $E(k) = 2\Delta - 2 \cos ka$   
 positions  $x, y$   $x < y$   
 Not adjacent  $E C_{x,y} = -C_{x+1,y} - C_{x-1,y} - C_{x,y+1} - C_{x,y-1} + 4 \Delta C_{x,y}$  (1)

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Ludwig Dimitrevich Faddeev:  
*"No, these are not what I saw!  
I saw a very thick notebook."*

Russian Academy of Sciences  
"Uspekhi Fizicheskikh Nauk"  
("Physics-Uspekhi") journal  
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E-mail: [keldysh@ufn.ru](mailto:keldysh@ufn.ru), [maria@ufn.ru](mailto:maria@ufn.ru)

January 30, 2013

To whom it may concern  
California Institute of Technology  
On-line Archive of California  
(OAC)

Dear Colleagues,

The Editorial Board of "Uspekhi Fizicheskikh Nauk" ("Physics-Uspekhi" in English version) journal (one of the leading Russian journal in physics, former Editor-in-Chief Nobel Prize Winner Vitaly Ginzburg) needs your kind permission for our scientific and managing editor *Dr. Maria Aksenteva* to get the opportunity to see some documents from your archive.

One of the world-recognized mathematician Prof. Ludvig Faddeed is now writing an interesting review for our journal. Several years ago, just after the death of Prof. Richard Feynmann, Prof. Faddeev has visited Caltech and saw some hand-written notes by Prof. R. Fenmann in which he emphasized the importance of Bethe Ansatz for some fundamental physical problems.

But it is impossible to investigate these folders via Internet.

Due to the fact, that Dr. Maria Aksenteva is going to visit California in February we hope, that it would be possible for her to look at these folders.

Thank you in advance for your kind help and support. We would greatly appreciate your kind positive reply.

With kind regards,  
on behalf of UFN Editorial Board  
Academician Rudenko O.V.  
Associate Editor of the "Uspekhi Fizicheskikh Nauk"  
("Physics-Uspekhi") journal ([www.ufn.ru](http://www.ufn.ru))



Faddeev decided to take  
it upon himself to find  
the missing notebooks.

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E-mail: [keldysh@ufn.ru](mailto:keldysh@ufn.ru), [maria@ufn.ru](mailto:maria@ufn.ru)

January 30, 2013

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On-line Archive of California  
(OAC)

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("Physics-Uspekhi") journal ([www.ufn.ru](http://www.ufn.ru))



Faddeev decided to take  
it upon himself to find  
the missing notebooks.

But, the notebook he saw  
was not at the Archives.

Where is the notebook  
Faddeev saw?

Elias Kiritsis (Caltech 1988 Ph.D.)

*When I arrived at Caltech in the fall of 1984, Feynman was "away" from Lauritsen. I was told that he was spending his time in the computer science department, as he was interested in "quantum computation", and his famous work on the subject dates from that era.*

*In 1986, he suddenly he reappeared in the Physics department with a new "hobby" that he immediately shared with all people. **His intuition was that to solve QCD you must define the right variables like in integrable models.** He started spending time with us, and having often lunch with us as he was trying to share his excitement and make us interested.*



Alexios Polychronakos (Caltech 1987 Ph.D.)

*Last time I saw Feynman was when I visited Caltech Dec 1987 - Jan 1988. He passed soon after that, and I did not visit Caltech again until sometime late in 1988, which was when his secretary, the late **Helen Tuck**, gave me a folder with his notes, thinking that I should have them. The folder is still in my possession.*

*Since Faddeev apparently saw the hand-written notes in question "just after the death of Prof. Feynman," there is a small chance that he might be referring to the ones I have, although they are on Calogero-Sutherland models and not on the Bethe ansatz.*



Sandip Trivedi (Caltech 1990 Ph.D.)

***Feynman's idea was to use the progress in integrability in two dimensions to study fragmentation or hadronisation in QCD.***

*Consider a high energy scattering process which gives rise to a quark or gluon moving at high energy that fragments as it moves away from the interaction region. There is one preferred direction along which the quark or gluon moves where the momentum is high and the physics is "hard", and two transverse directions where it is "soft".*

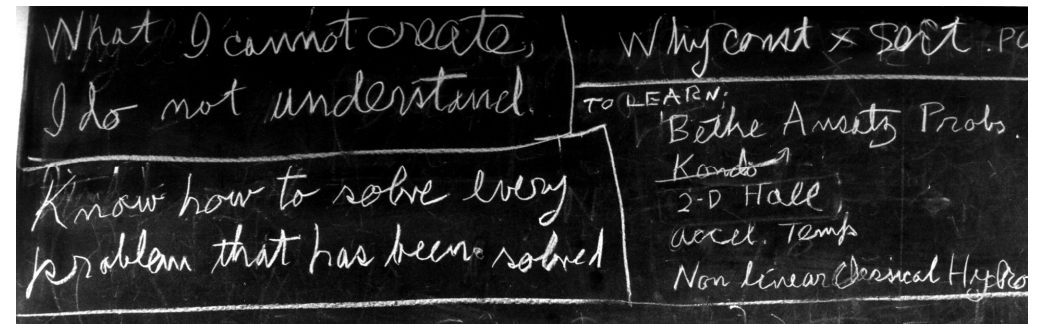
***Feynman's idea was to separate these directions. Since high energy physics is simple in QCD due to asymptotic freedom, the hard physics would be easy to deal with. This would then leave the two transverse directions where the physics is soft and non-trivial, and perhaps here the progress in integrability could be used to understand the process of fragmentation.***

Sandip Trivedi (Caltech 1990 Ph.D.)

*The first and perhaps most important lesson, for a beginning graduate student like me, was just how thorough and careful Feynman was.*

***Feynman explained the six-vertex model to us, and his instructions were for us to try and solve it without looking at Baxter's solution, already in print.***

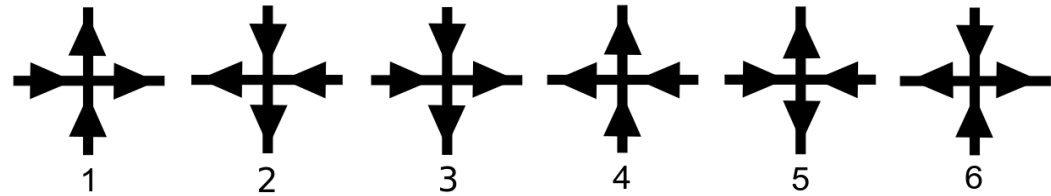
*"We gotta know how to solve every problem that has been solved."*



*Occasionally the discussions would extend over dinner - for which Feynman often generously paid. I remember one great dinner at a Mexican restaurant on Colorado Boulevard where he explained the spin statistics theorem, using a belt which he took off from his pant.*

Sandip Trivedi (Caltech 1990 Ph.D.)

On the six-vertex model:



*One day Feynman called us in to his office. His eyes were shining.*

*“I have solved it !” he exclaimed.*

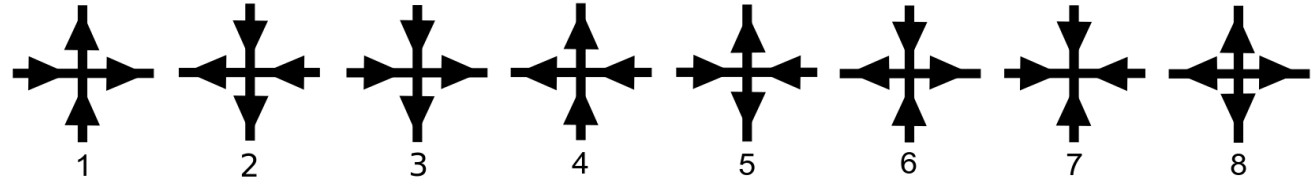
*“Get me some multi-coloured chalk !” he yelled to his secretary Helen.*

*What followed was pure Feynman genius, in equal measure physics and showmanship of the highest order.*

*The key was to look at the lattice not in the standard fashion, as we had been doing, but at 45 degrees. Then, due to the particle number conservation, a solution easily presented itself in a few lines of algebra.*

Sandip Trivedi (Caltech 1990 Ph.D.)

On the eight-vertex model:



*This was more complicated because there was no particle number conservation, and particles could be created and destroyed. Feynman was optimistic as ever. "I invented all the tricks on how to deal with the creation and destruction of particles in QED," he said to us with a wink. "We will surely get there".*

*Sadly, Feynman fell quite ill soon thereafter. Our meetings grew infrequent.*

Sandip Trivedi (Caltech 1990 Ph.D.)

*I do have some notes of Feynman which are a treasured possession of mine. During his final illness, **Feynman instructed Helen Tuck to share his notes with the students.** We were amazed how meticulous and detailed they are. I show the notes to my students to inspire them.*

*When I look back one of the things that strikes me is just how remarkable a place Caltech was, and continues to be.*

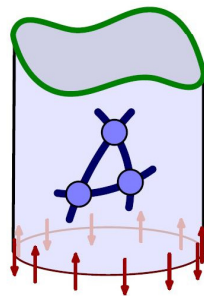
*I could come in as a beginning graduate student and soon thereafter start working with one of the greatest scientists of the 20th century. Where else does one get an opportunity like this, so easily!*

# Feynman's vision is being partially realized in supersymmetric gauge theories.

## Review of AdS/CFT Integrability: An Overview

NIKLAS BEISERT<sup>†1</sup>, CHANGRIM AHN<sup>2</sup>, LUIS F. ALDAY<sup>3,4</sup>, ZOLTÁN BAJNOK<sup>5</sup>, JAMES M. DRUMMOND<sup>6,7</sup>, LISA FREYHULT<sup>8</sup>, NIKOLAY GROMOV<sup>9,10</sup>, ROMUALD A. JANIK<sup>11</sup>, VLADIMIR KAZAKOV<sup>12,13</sup>, THOMAS KLOSE<sup>8,14</sup>, GREGORY P. KORCHEMSKY<sup>15</sup>, CHARLOTTE KRISTJANSEN<sup>16</sup>, MARC MAG TRISTAN McLOUGHLIN<sup>1</sup>, JOSEPH A. MINAHAN<sup>8</sup>, RAFAEL I. NEPOMECHI ADAM REJ<sup>19</sup>, RADU ROIBAN<sup>20</sup>, SAKURA SCHÄFER-NAMEKI<sup>9,21</sup>, CHRISTOPH SIEG<sup>22,23</sup>, MATTHIAS STAUDACHER<sup>22,1</sup>, ALESSANDRO TORRIE ARKADY A. TSEYTLIN<sup>19</sup>, PEDRO VIEIRA<sup>26</sup>, DMYTRO VOLIN<sup>20</sup> AND KONSTANTINOS ZOUBOS<sup>16</sup>

<sup>†</sup>corresponding author, e-mail address: nbeisert@aei.mpg.de



## Long-range $\mathfrak{psu}(2, 2|4)$ Bethe ansätze for gauge theory and strings

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In Honor of Hans Bethe

## Gauge Theory And Integrability, I

Kevin Costello<sup>1</sup>, Edward Witten<sup>2</sup> and Masahito Yamazaki<sup>3</sup>

## Gauge Theory And Integrability, II

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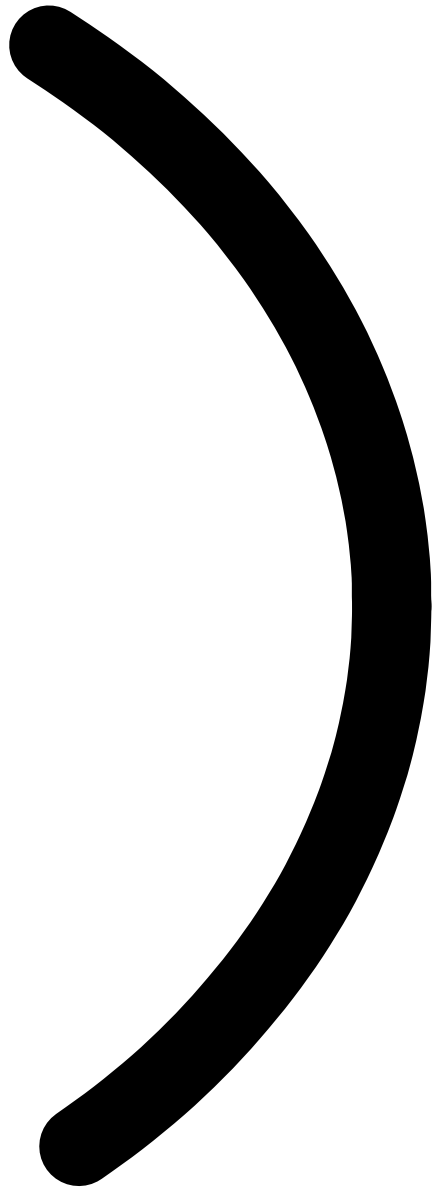
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Several  
equation as  
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## Abstract

Starting with a four-dimensional gauge theory approach to rational, elliptic, and trigonometric solutions of the Yang-Baxter equation, we determine the corresponding quantum group deformations to all orders in  $\hbar$  by deducing their RTT presentations. The arguments we give



end of the digression

# Is non-renormalizability a problem with gravity?





It is often said that, since the Einstein theory,

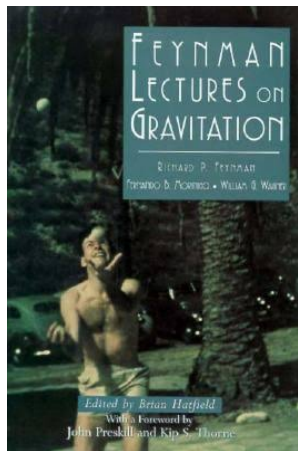
$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \Lambda + \frac{1}{G_N} \mathcal{R} + \text{matter} \right)$$

is ***not renormalizable*** and cannot be used to compute quantum gravity effects reliably.

It is often said that, since the Einstein theory,

$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \Lambda + \frac{1}{G_N} \mathcal{R} + \text{matter} \right)$$

is ***not renormalizable*** and cannot be used to compute quantum gravity effects reliably.



*"Whether it is a truly significant objection to a theory, I don't know."*

Feynman Lectures on Gravitation (1962 - 63)

However, ....



Kenneth G. Wilson (1936 - 2013)

1961 **Caltech** PhD

1982 Nobel Prize

For example, the pion theory in nuclear physics,

$$\mathcal{S} = \int d^4x \frac{(\partial \vec{\pi}(x))^2}{1 + \vec{\pi}(x)^2 / F^2}$$

is also *not renormalizable*.

Nevertheless, we can use it to calculate quantum effects reliably as far as energy scale is less than  $F$ , and they have been verified experimentally.

It is an **effective theory** of the more fundamental QCD.

John Preskill:

*I spoke with Feynman a number of times about renormalization theory during the mid-80s. **I was surprised how the effective field theory viewpoint did not come naturally to him.***

*Mark Wise gave a talk about chiral Lagrangian, and Feynman thought Mark was foolishly doing perturbation theory in the pion-nucleon scattering. I explained that the calculation was really an expansion in the pion momentum, and therefore justified when the pions are sufficiently soft. He understood quickly, but I was surprised he needed my prompting to catch on.*

*It was funny in a way. I would tell Feynman things that I knew were common lore, and he would be receptive, but **he seemed unfamiliar with the ideas.***

Sidney Coleman:

*[Feynman] was like the guy that climbs Mt. Blanc barefoot just to show it could be done. A lot of things he did were to show, you didn't have to do it that way, you can do it this other way. And the other way, in fact, was not as good as the first way, but it showed he was different.*

*I am sure Dick thought of that as a virtue, as noble. I don't think it's so. I think it's kidding yourself. Those other guys are not all a collection of yo-yos. **Sometimes it would be better to take the recent machinery they have built and not try to rebuilt it, like reventing the wheel.***

*I know people who are in fact very original and not cranky but have not done as good physics as they could have done because **they were more concerned at a certain juncture with being original than being right.** Dick could get away with a lot because he was so goddam smart. He really could climb Mt. Blanc barefoot.*

from "Genus" by James Gleick

# Einstein gravity is also an **effective theory**.

As in the pion theory, the Einstein gravity can be used to make reliable predictions including quantum effects, provided energy and momenta are below its cutoff scale (threshold above which a more fundamental theory, such as string theory, is required).

- For example:
- ☆ Hawking radiation from a black hole
  - ☆ Cosmic microwave background fluctuations caused by quantum effects during the inflation
  - ☆ Corrections to the Newton potential:

$$V = -\frac{G_N m_1 m_2}{r} \left( 1 - \frac{G_N (m_1 + m_2)}{r} - \frac{135}{30\pi^2} \frac{G_N \hbar}{r^2} + \dots \right)$$

relativity effect

quantum effect

However, ....

**Gravity is Different.**



# Gravity is Different:

- (1) Not all low energy theories of gravity can be derived from consistent quantum theories.
- (2) The physical world is hierarchial. The exploration of shorter distances has led us to more and more fundamental laws of nature. This hierarchy will terminates once we complete quantum gravity.



Remarkably, there is a theory which successfully quantize gravity and which contains all ingredients for a realistic model of particle physics.

## *Superstring Theory*

Michael Green and John Schwarz:

*Crucial breakthroughs were made in August 1984 at the Aspen Center for Physics.*

*While walking to one of the workshop seminars, JHS remarked to MBG that there might be a gauge group for which the two contributions cancel. At the end of the seminar MBG said to JHS "SO(32)," which was the correct result.*

ACP Science Histories



The 20th anniversary  
Celebration in Aspen  
on 12 August 2004

# Swampland Question

Given an effective theory of gravity, how can one judge whether it is realized as a low energy approximation to a consistent quantum theory such as string theory?

Vafa: hep-th/0509212;

Vafa + Ooguri: hep-th/0605264

# Constraints on Symmetry

Symmetry has played important roles in physics

- (1) In identifying and formulating fundamental laws of nature
- (2) In using these laws to understand and predict dynamics and phases of matters.

Symmetry can be deceiving:

Two seemingly different microscopic Lagrangians with **different gauge symmetries** and different matter contents **can describe the same quantum system.**

**"Duality"**

Equivalences can be between full Hilbert spaces or about their low energy limits (such as in the Seiberg dualities).

Symmetry can be deceiving:

**Global symmetry** is well-defined and is independent of which Lagrangian description you use.

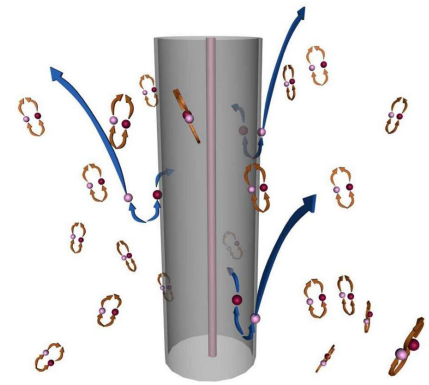
Symmetry can be deceiving:

**Global symmetry** is well-defined and is independent of which Lagrangian description you use.

However, it has been argued that a **consistent quantum theory of gravity does not have global symmetry.**



# Standard argument against global symmetry in quantum gravity:



If there is a continuous global symmetry  $G$ , we can combine a large number of  $G$ -charge matters to make a **black hole in an arbitrary large representations of  $G$** .

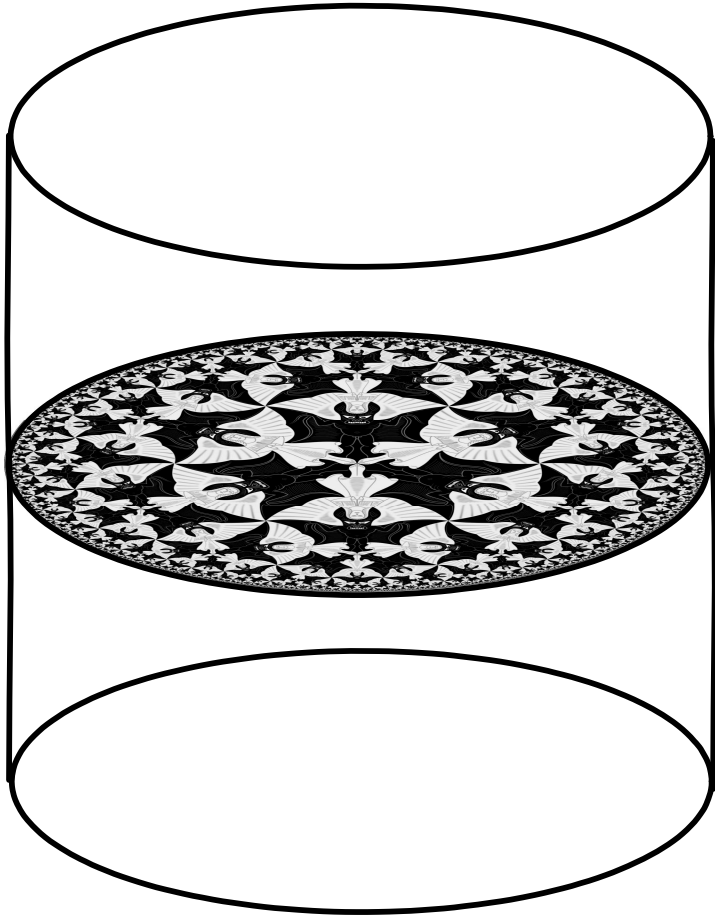
**Let it Hawking-radiate**, keeping its mass  $>$  the Planck mass.

The Hawking radiation is  $G$ -blind (If  $G$  were a gauge symmetry, the radiation would have charge imbalance). The dimension of the  $G$  representation exceeds **the number of states allowed by the Bekenstein-Hawking entropy** formula.

We have refined and proven these conjectures in AdS/CFT.

《 work in progress with Daniel Harlow 》

- (1) Any **global symmetry** in AdS is **inconsistent** with locality of CFT.
- (2) A compact (discrete or continuous) symmetry  $G$  in CFT corresponds to a **gauge symmetry** with the same  $G$  in AdS.
- (3) In a gravitational theory with gauge group  $G$ , there must be physical states in **every finite dimensional irreducible unitary representation** in  $G$ .
  - + with some additional assumption:
    - (4) Internal global symmetry of CFT is compact.

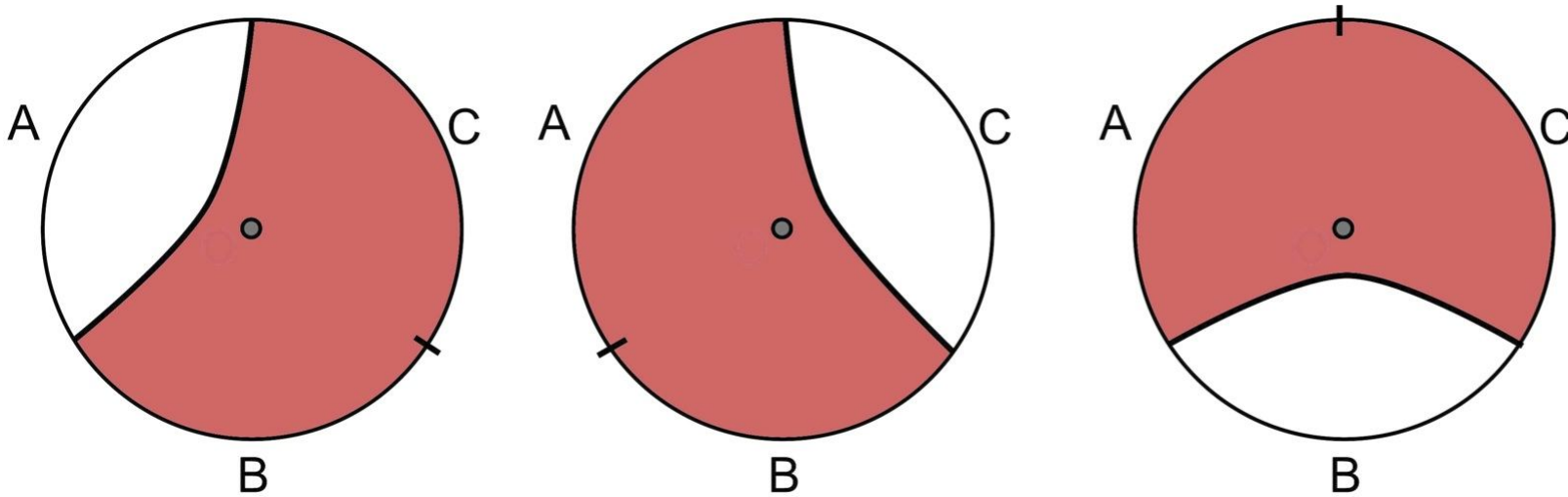


## Holographic Principle:

Gravitational theory in **Anti-de Sitter** space (AdS) is equivalent to **Conformal Field Theory** (CFT) at the boundary.

$$\text{AdS} = \text{CFT}$$

The correspondence enables us to prove theorems about quantum gravity by translating them into statements on conformal field theory.



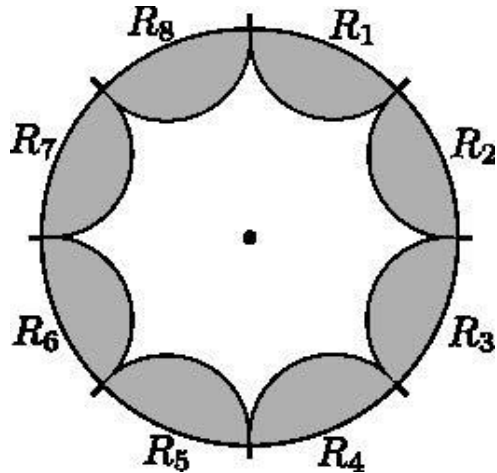
Recently, it was discovered that local excitations in the gravitational theory have a special type of quantum entanglement, similar to the one used for **quantum error correcting codes**, with sub-systems sharing **quantum secret keys**.

# Quantum gravity cannot have symmetry

If a gravitational theory in has symmetry,

# Quantum gravity cannot have symmetry

If a gravitational theory in has symmetry, there must be an operator transforming a local excitation into another.



However, a symmetry generating operator **cannot change quantum error correcting properties of quantum states.**

**Contradiction**

# Weak Gravity Conjecture





# Weak Gravity Conjecture

If a low energy theory contains both gravity and electromagnetism, the theory must also contain a particle with charge  $Q$  and mass  $m$ , such that:

$$m \leq \frac{|Q|}{\sqrt{G}}, \quad G : \text{Newton constant}$$

Arkani-Hamed, Motl, Nicolis, Vafa: hep-th/0601001

In all cases,

$$m < \frac{Q}{\sqrt{G}} \quad (\text{no "="}) \quad \text{unless protected by supersymmetry}$$

We proposed this as a stronger version of the conjecture and showed that it **restricts types and masses of neutrinos**

Vafa + Ooguri: 1610.1533

Our idea has been explored further in recent papers, leading to **constraints on particle physics models beyond the Standard Model.**

Ibanez, Martin-Lozano, Valenzuela: 1706.05392, 1707.05811

Hamada, Shiu: 1707.06326

Gonzalo, Herraez, Ibanez: 1803.08455

# Are de Sitter vacua in the Swampland?

The Dine-Seiberg problem (1985) poses a question to any attempt to construct of meta-stable vacua with zero or positive vacuum energy in the weakly coupled regime of string theory. The problem can be sharpened by applying recent insights from string theory [to appear with E. Palti, G. Shiu, C. Vafa].

SPACE

## String Theory May Create Far Fewer Universes Than Thought

Some physicists claim the popular landscape of universes in string theory may not exist

By Clara Moskowitz on July 30, 2018

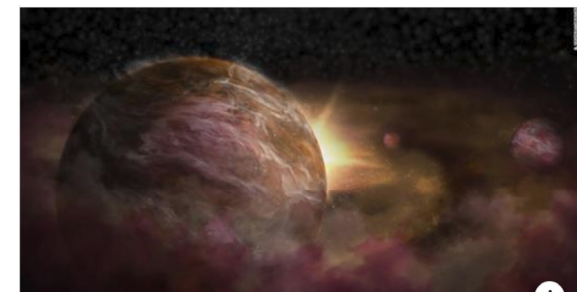


SCIENCE

## The Universe as We Understand It May Be Impossible

A new conjecture in physics challenges the leading “theory of everything.”

NATALIE WOLCHOVER AND QUANTA AUG 13, 2018



CNN.COM

The scientific theories battling to explain the universe

The theories of superstrings and dark energy give us insight into the...

**Without gravity**, any low energy effective theory can be embedded in a consistent high energy theory.

**With gravity**, the situation is different, and there are **non-trivial constraints** on gravitational low energy effective theories (swampland conditions).

With more work, we may be able to extract predictions on observable phenomena in low energy from consistency of quantum gravity in high energy.

*"And so I 'm stuck to have to continue this investigation, and of course you all appreciate that this is the secret reason for doing any work, no matter how absurd and irrational and academic it looks; we all realize that no matter how small a thing is, if it has physical interest and is thought about carefully enough, you're bound to think of something that is good for something else."*

based on a tape-recording of Feynman's lecture  
"Quantum Theory of Gravitation" in Poland in July 1962.