When $^{\exists}$exactly marginal operators

Hirosi Ooguri

“Interface & Symmetry”
2 – 6 March 2026, Yukawa Institute, Kyoto University

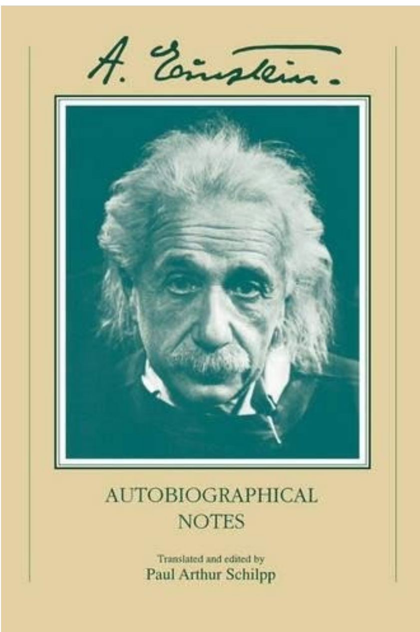
Four years ago…

Perspectives and Prospects

Hirosi Ooguri

Strings Vienna, 2022
18-22 July

An earlier formulation:



Autobiographical Notes
by Albert Einstein

After explaining the notion of the natural units,

"…, then **only dimensionless constants could occur in the basic equations of physics**. Concerning such I would like to state a theorem which at present cannot be based upon anything more than upon a faith in the simplicity, *i.e.*, intelligibility, of nature: **there are no arbitrary constants of this kind** …"

A modern formulation:

Vafa + H.O.: 0605264

Every parameter in quantum gravity is an expectation value of a **dynamical field** and can be varied by changing its expectation value.

Can we prove this statement in AdS/CFT?

Strings 2022 Summary Talk

Every parameter in quantum gravity is an expectation value of a **dynamical field** and can be varied by changing its expectation value.

If there is a **parameter in the AdS Lagrangian**, there must be a corresponding parameter in the dual CFT.

If the CFT parameter can be deformed by adding an exactly marginal operator to the CFT Lagrangian, there must be a **dynamical field** in AdS.

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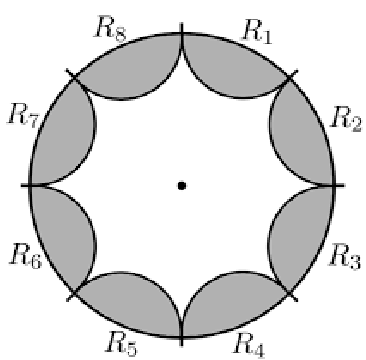
This conditional clause is a conjecture in CFT.

Can every CFT parameter λ be deformed by adding an exactly marginal operator ϕ to the CFT Lagrangian?

$$S_{\mathrm{CFT}} \longrightarrow S_{\mathrm{CFT}} + \delta\lambda \int_{\mathbb{R}\times\Sigma} \phi$$

This is analogous to the Noether theorem,

Splittablity: $U(g,\Sigma) = \prod_i U(g,\Sigma_i)$ when $\Sigma = \cup_i \Sigma_i$,



which was used to prove no global symmetry in quantum gravity.

Harlow, H.O.: 1810.05338

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This can be regarded as the Noether theorem for a (-1)-form symmetry.

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This can be regarded as the Noether theorem for a (-1)-form symmetry.

No parameter in quantum gravity $\Longleftrightarrow$ **No** (-1)-form **global symmetry** in quantum gravity

- No adjustable parameters (1949)

QUANTIFY →

Distance Conjecture (2006)

$$m = \exp(-\alpha\,\phi + O(1))$$

- No global symmetry (1957)

QUANTIFY →

Weak Gravity Conjecture (2006)

$$m^2 \leq \frac{d-2}{8\pi G_{\mathrm{N}}(d-3)}\,Q^2$$

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Distance Conjecture (2006)

$$m = \exp(-\alpha\,\phi + O(1))$$

Unified

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QUANTIFY →

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Strings 2022 Summary Talk

Every parameter in quantum gravity is an expectation value of a **dynamical field** and can be varied by changing its expectation value.

If there is a **parameter in gravity** in AdS, there must be a corresponding parameter in the dual CFT.

If the CFT parameter can be deformed by adding an exactly marginal operator to the CFT Lagrangian, there must be a **dynamical field** in AdS.

This conditional clause is a conjecture in CFT.

The question for today:

If exactly marginal operators exist in a CFT, they generate a conformal manifold, parameterizing a continuous family of CFTs. Does the converse also hold? Namely, **does a conformal manifold require the existence of exactly marginal operators?**

$$S_{\mathrm{CFT}} \longrightarrow S_{\mathrm{CFT}} + \delta\lambda \int_{\mathbb{R}\times\Sigma} \phi$$

We answer this question affirmatively, with the following three assumptions:

Komatsu, Kusuki, Meineri + H.O.: 2512.11045

1. $^{\exists}$**Interface:** For any pair of CFTs that are close to each other on the conformal manifold, there is a conformal interface that becomes trivial when the two CFTs coincide.
2. **Smoothness:** Correlators involving unit normalized defect operators have finite limits as the interface disappears.
3. **Technical assumptions (?):** The bulk CFT has no operators with conformal weights of (½, ½) that are neutral under all the global symmetries preserved by the interface.

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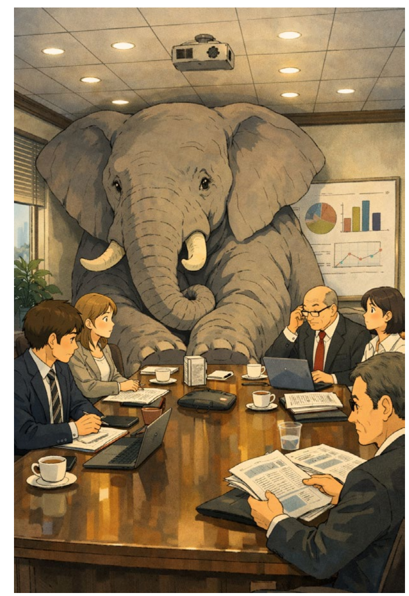
2. **Smoothness:** Correlators involving unit normalized defect operators have finite limits as the interface disappears.

These two assumptions imply that **defect operators become local operators of the CFT in the limit**, unless they decouple.

We answer this question affirmatively, with the following two assumptions:

3. Technical assumptions (?): The bulk CFT has no operators with conformal weights of (½, ½) that are neutral under all the global symmetries preserved by the interface.

This assumption exclude the **Liouville theory** since the theory contains a symmetry-neutral (½, ½) operator. The operator may not correspond to a normalizable state in the Liouville theory, but our argument is insensitive to the normalizability issue.



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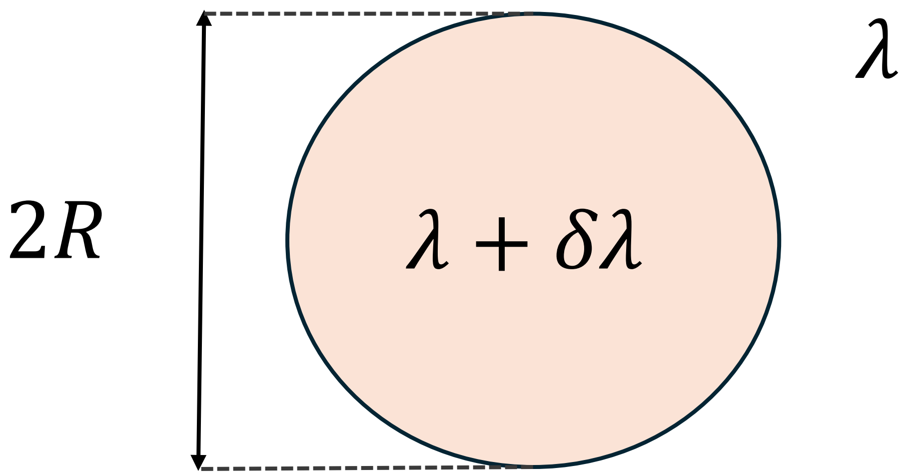
3. Technical assumptions (?): The bulk CFT has no operators with conformal weights of (½, ½) that are neutral under all the global symmetries preserved by the interface.

However, that is not why we need this additional assumption.

Intuitive idea: $\langle \cdots \ \phi \rangle = \lim_{R,\delta\lambda \to 0} \frac{1}{\mathrm{Vol}_R} \frac{\partial}{\partial \lambda} \langle \cdots \ \phi \rangle$



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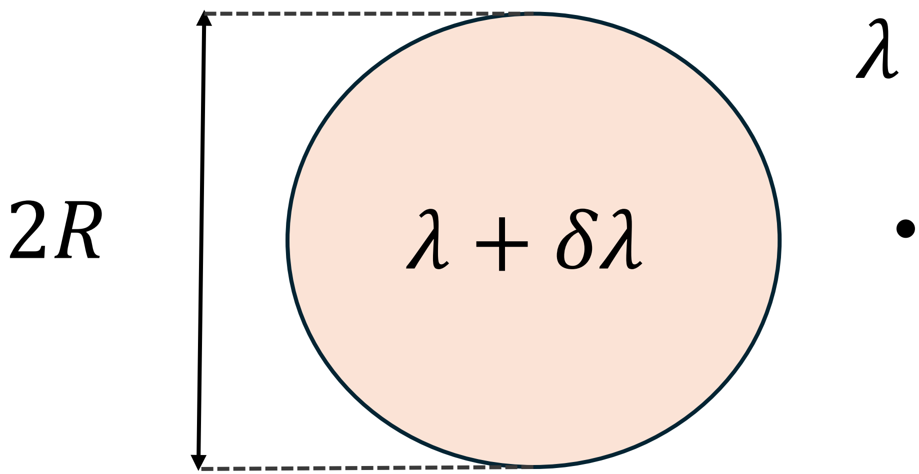


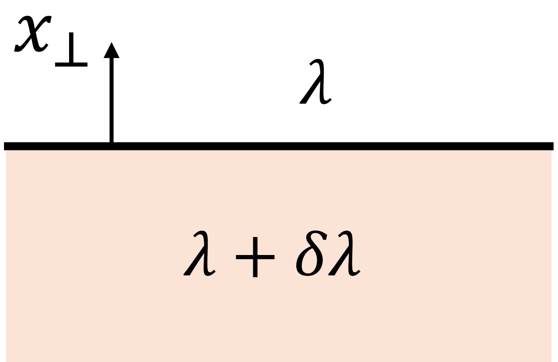
- How should the limit be taken, including the order of $R \to 0$ and $\delta\lambda \to 0$?

- Is the limit non-zero and finite?
- What is the role of the stress-energy tensor $T_{\mu\nu}$? Otherwise, generalized free fields and QFTs without gravity in AdS become counter-examples

For the next few pages, we will restrict our attention to $d = 2$.

We will come back to $d \geq 2$ later.

Displacement operator D: $\partial_\mu T^{\mu\perp} = \delta(x_\perp) D(x_\parallel)$



$$\langle D(x) D(y) \rangle = \frac{N_D}{|x-y|^{2\Delta_D}}$$

$$\Delta_D = d (= 2)$$

$$N_D = 2(c_1 + c_2 - 2c_{12}) \propto \text{(reflection coefficient)}$$

If the CFT deformation is generated by an exactly marginal operator ϕ as $S_{\mathrm{CFT}} \longrightarrow S_{\mathrm{CFT}} + \delta\lambda \int \phi$,

$T(x + i0) - T(x - i0) = \pi\delta\lambda\phi(x) + O(\delta\lambda^2)$ on the interface

and $N_D = (2\pi\delta\lambda)^2 + O(\delta\lambda^3)$.

Just above and below the interface, let us write

$T_1(x) = T(x + i0)$, $T_2(x) = T(x - i0)$ and define

$\widehat{D}(y) = D(x)/\sqrt{N_D}$ so that $\langle D(x)D(y)\rangle = |x - y|^{-2\Delta_D}$.

By the Ward-Takahashi identities,

$$
\left(T_1(x) + \bar{T}_2(x)\right) \widehat{D}(y)
= \frac{(c_1 - c_2)/\sqrt{N_D}}{(x - y)^4} + \left(\frac{2}{(x - y)^2} + \frac{1}{x - y}\partial\right)\widehat{D}(y) + O(1)
$$

(Note that we do **not** assume $c_1 = c_2$. Due to the quantization of $c - \bar{c}$, the existence of the conformal interface requires the weaker condition, $c_1 - \bar{c}_1 = c_2 - \bar{c}_2$.)

$$\left(T_1(x) + \bar{T}_2(x)\right) \widehat{D}(y) = \frac{(c_1 - c_2)/\sqrt{N_D}}{(x-y)^4} + \left(\frac{2}{(x-y)^2} + \frac{1}{x-y}\partial\right)\widehat{D}(y) + O(1)$$

Note $N_D = 2(c_1 + c_2 - 2c_{12}) \geq 2|c_1 - c_2|$ because of the null energy condition.* Therefore, in the limit where the interface becomes transparent, $(c_1 - c_2)/\sqrt{N_D} \longrightarrow 0$ and $\widehat{D}$ becomes an interface primary of weight 2. In particular, $\widehat{D}$ does not decouple in the limit.

By the smoothness assumption, the defect operator $\widehat{D}$ should become a bulk local operator in the limit where the interface disappears.

* Meineri, Penedones, Rousset:1904.10974

If $\widehat{D}$ becomes a Virasoro primary in the limit,

$$\left(T(x)-\bar{T}(x)\right)\widehat{D}(y)=\frac{h-\bar{h}}{(x-y)^2}\widehat{D}(y)+O(1).$$

Since $\widehat{D}\propto T-\bar{T}$, its OPE with $(T-\bar{T})$ should be non-singular.

Therefore, $h=\bar{h}=1$.

If $\widehat{D}$ contains Virasoro descendants, the only possibility is $\left(\partial-\bar{\partial}\right)\rho$ for some (½, ½) operator ρ. Since $\widehat{D}$ is invariant under any global symmetry, so is ρ. By the assumption 3, such an operator does not exist. Therefore, $\widehat{D}$ becomes a marginal operator ϕ in the limit.

$$\lim_{N_D\to 0}\widehat{D}=\phi\text{: marginal}$$

Is $\lim_{N_D \to 0} \widehat{D} = \phi$ **exactly** marginal?

Suppose $\mathcal{O}_1(z, \bar{z})$ and $\mathcal{O}_2(\bar{z}, z)$ are quasi-primaries with $h_1 - \bar{h}_1 = h_2 - \bar{h}_2$ located at antipodal points across the interface. With and without the displacement operator,

$$\langle \mathcal{O}_1(z, \bar{z}) \mathcal{O}_2(\bar{z}, z) \rangle = \frac{a_{12}}{(2 \operatorname{Im} z)^{\Delta_1 + \Delta_2}} \quad (1)$$

$$\left\langle \mathcal{O}_1(z, \bar{z}) \mathcal{O}_2(\bar{z}, z) \widehat{D}(w) \right\rangle = \frac{(\Delta_1 - \Delta_2) a_{12} / \sqrt{N_D}}{(2 \operatorname{Im} z)^{\Delta_1 + \Delta_2 - 2}} \quad (2)$$

Setting $\mathcal{O}_1 = \mathcal{O}_2 = \phi$, the RHS of (2) vanishes identically for any N_D, while the RHS of (1) does not and in particular $\lim_{N_D \to 0} a_{12} = 1$. Therefore,

$$\langle \phi(z, \bar{z}) \phi(\bar{z}, z) \phi(w) \rangle = 0.$$

More generally, with $\mathcal{O}_1$ and $\mathcal{O}_2$ two operators such that $\mathcal{O}_2 \to \mathcal{O}_1$ in the limit $N_D \to 0$, the two equations

$$\langle \mathcal{O}_1(z,\bar{z})\mathcal{O}_2(\bar{z},z)\rangle = \frac{a_{12}}{(2\ \mathrm{Im}\ z)^{\Delta_1+\Delta_2}} \qquad (1)$$

$$\left\langle \mathcal{O}_1(z,\bar{z})\mathcal{O}_2(\bar{z},z)\widehat{D}(w)\right\rangle = \frac{(\Delta_1-\Delta_2)a_{12}/\sqrt{N_D}}{(2\ \mathrm{Im}\ z)^{\Delta_1+\Delta_2-2}} \qquad (2)$$

reproduce the conformal perturbation equation,

$$\delta\Delta = \sqrt{N_D}\, c_{\mathcal{O}\mathcal{O}\phi} + o\left(\sqrt{N_D}\right).$$

Can't we relax the technical assumption?

"The bulk CFT has no operators with conformal weights of (½, ½) that are neutral under all the global symmetries preserved by the interface."

Without this assumption, in general $\lim_{N_D \to 0} \widehat{D} = \phi + \left(\partial - \bar{\partial}\right)\rho$.

How do we know that $\phi \neq 0$?

If $\phi = 0$, we can prove $c_{\rho\rho\rho} = 0$. A line defect given by integrating ρ is a part of the conformal manifold, e.g., the Oshikawa-Affleck defect of the Ising CFT.

Even though ρ is unrelated to bulk moduli, we have not been able to exclude the possibility of $\lim_{N_D \to 0} \widehat{D} = \left(\partial - \bar{\partial}\right)\rho$.

For example, what if $D = \sqrt{N_D}\left({N_D}^{\varepsilon}\phi + \left(\partial - \bar{\partial}\right)\rho\right)$ with $\varepsilon > 0$?

Generalization to $d \geq 2$

In two dimensions, we found $\lim_{N_D \to 0} \widehat{D} = \phi + (\partial - \bar{\partial})\rho$,
where $\Delta_\phi = 2$ and $\Delta_\rho = 1$.

This is generalized to $d \geq 2$ as

$$\lim_{N_D \to 0} \widehat{D} = \phi + \sum_{n=1}^{\left\lfloor \frac{d}{2} + 1 \right\rfloor} \partial_\perp^n \rho_n + \text{(parallel derivatives)},$$

where is a scalar quasi-primary of dimension $\Delta_\phi = d$ and ρ_n's are scalar quasi-primaries with dimension $\Delta_{\rho_n} = d - n$.

If $\phi \neq 0$, the argument in the $2d$ case can be generalized to show $c_{\phi\phi\phi} = 0$

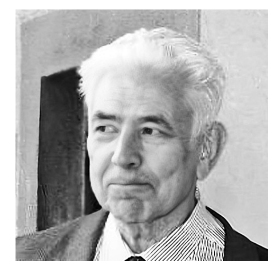
(Obvious) **questions:**

- Can we remove the technical assumption on ρ or are there counter-examples without the assumption?

- Can we derive the existence of the conformal interface?

If you can't prove your theorem, keep shifting parts of the conclusion to the assumptions, until you can.

attributed to Ennio De Giorgi



This work was partially motivated by

Conjecture: Every parameter in quantum gravity is an expectation value of a dynamical field and can be varied by changing its expectation value.

This by itself **does not provide a useful constraint** since what appear to be adjustable parameters in a low energy theory may be fixed by potentials in a more fundamental high energy theory.

For example, the Standard Model of Particle Physics has 19 parameters (plus 7 with massive neutrinos) and the ΛCDM Model of Cosmology has 6 parameters; They do not contain dynamical fields corresponding to these parameters.

Distance Conjecture

Vafa + **H.O.**: 0605264

Conjecture 0: Every parameter in quantum gravity is an expectation value of a dynamical field and can be varied by changing its expectation value.

Conjecture 1: Choose any point p_0 in the moduli space $\mathcal{M}$. For any positive ϕ, there is another point $p \in \mathcal{M}$ such that $d(p, p_0) > \phi$.

Conjecture 2: Compared to the theory at $p_0 \in \mathcal{M}$, the theory at p with $d(p, p_0) > \phi$ has an infinite tower of light particles starting with mass of the order of $e^{-\alpha\phi}$ for some $\alpha > 0$.

We have proven a part of Distance Conjecture in $\mathrm{AdS}_3/\mathrm{CFT}_2$ using the conformal bootstrap.

Wang + H.O.: 2405.00674

If there is a particle in AdS_3 whose mass vanishes at some point in the moduli space, the distance σ to the point is infinite, the mass vanishes exponentially,

$$m^2 = \frac{s-1}{\ell_{\mathrm{AdS}}^2} \exp(-\boldsymbol{\alpha}\, \sigma + O(1))$$

and **α is bounded as,**

$$\left(\frac{2}{3}\ell_{\mathrm{P}}\right)^{1/2} \leq \boldsymbol{\alpha} \leq (8\pi\ell_{\mathrm{AdS}})^{1/2}$$

Planck scale

AdS scale

If CFT_2 has a holographic dual in AdS_3

$$m^2 = \frac{s-1}{\ell_{\mathrm{AdS}}^2} \exp(-\alpha_{\mathrm{AdS}}\, \sigma + O(1))$$

$$\left(\frac{2}{3}\ell_{\mathrm{P}}\right)^{1/2} \leq \alpha_{\mathrm{AdS}} \leq (8\pi\ell_{\mathrm{AdS}})^{1/2}$$

- When $\sigma \geq \left(\frac{2}{3}\ell_{\mathrm{P}}\right)^{-\frac{1}{2}}$, the tower of light particles **must emerge**.
- When $\sigma \geq (8\pi\ell_{\mathrm{AdS}})^{-\frac{1}{2}}$, the tower of light particles **can emerge**.

Question:

Can we formula and derive these conjectures in CFT?

- No adjustable parameters (1949)

QUANTIFY ⇒ Distance Conjecture (2006)

$$m = \exp(-\alpha\,\phi + O(1))$$

- No global symmetry (1957)

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